Economics: A Biophysical Theory

Jing Chen and James Galbraith
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Abstract

Most people agree that human activities are consistent with physical laws. One may naturally think that sensible economic theories can be derived from physical laws and evolutionary principles. This is indeed the case. In this paper, we present a newly developed production theory of economics from biophysical principles. The theory is a compact analytical model that provides, in our view, a much more realistic understanding of economic (as well as social and biological) phenomena than the neoclassical theory of production. It greatly reduces the complexity of our understanding about the economic activities.

Keywords: biophysical economics, production theory, fixed cost, variable cost, return

Jing Chen
School of Business
University of Northern British Columbia
Prince George, BC
Canada V2N 4Z9
Phone: 1-250-960-6480
Email: chenj@unbc.ca
Web: http://web.unbc.ca/~chenj/

James K. Galbraith
Lyndon B. Johnson School of Public Affairs
The University of Texas at Austin
Austin, TX 78713-8925, USA
Email: galbraith@mail.utexas.edu
Web: http://www.utexas.edu/lbj/faculty/james-galbraith/

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1. Introduction

Most people agree that human activities are consistent with physical laws. One may naturally think that sensible economic theories can be derived from physical laws and evolutionary principles. This is indeed the case: an analytical thermodynamic theory of economics provides a more realistic and intuitive understanding of economic, social and biological phenomena than the mainstream economic theory (Chen, 2005; Chen and Galbraith, 2011, 2012a, 2012b). In this paper, we present a detailed discussion of production theory, which forms one part of the larger analytical theory. We will show how a biophysical production theory provides a simple and systematic understanding of investment activities and economic policies including taxation and regulation.

There are two fundamental properties of life. First, living organisms need to extract resources from the environment to compensate for the continuous diffusion of resources required to maintain various functions of life (Schrodinger, 1944). Second, for an organism to be viable, the total cost of extracting resources has to be less than the amount of resources extracted (Odum, 1971, Hall et al., 1986, Rees, 1992). The second property may be understood as natural selection rephrased as an economic principle. The purpose of our work is to show that a self-contained mathematical theory of production can be derived from these two fundamental properties of life.

From a physics perspective, resources can be regarded as low entropy sources (Georgescu-Roegen, 1971). The entropy law states that systems tend toward higher entropy states spontaneously. Living systems, as non-equilibrium systems, need to extract low entropy sources from the environment to compensate for their continuous dissipation. Such systems can be represented mathematically by lognormal processes, which contain both a growth term and a dissipation term. According to the entropy law, the thermodynamic dissipation of an organic or economic system is spontaneous. However, the extraction of low entropy sources from the environment depends on specific biological or institutional structures that incur fixed or maintenance costs. Additional variable cost is also required for resource extraction. Higher fixed cost systems generally have lower variable costs. Fixed cost is largely determined by the genetic structure of an organism or the design of a project. Variable cost is a function of the environment. An organism survives if the amount of resources it extracts is higher than the total cost spent. Similarly, a business survives if its revenue is higher than the total cost of production.

Product value, fixed cost, variable cost, discount rate, uncertainty, project duration and volume of output are major factors in determining the viability and success of production. These factors naturally became the center of investigation in early economic literature. However, because of the difficulty in forming a compact mathematical model about these factors, discussion about them became peripheral in current economic literature. With the help of the analytical production theory presented here, theoretical investigation in economics may refocus on these issues.

We set the initial condition so that total cost is equal to the amount of resource extracted or revenue generated. Then we derive a formula for variable cost as a mathematical function of product value, fixed cost, uncertainty, discount rate and project duration. From this formula, together with fixed cost and volume of output, we can compute and analyze the returns and profits of different production systems under various environmental conditions in a simple and systematic way. The results are highly consistent with empirical evidence obtained from the vast literature in economics and ecology. Furthermore, by putting major factors of production into a compact mathematical model, the theory provides precise insights about the tradeoffs and constraints of alternative business or evolutionary strategies.

We begin with the crucial relation between fixed cost and variable cost. Useful energy comes from the differential or gradient between two parts of a system. In general, the higher the differential, the more efficient the work becomes. At the same time, it is more difficult to maintain a system with a high differential. In other words, a lower variable cost system requires higher fixed cost to maintain it. This is a general principle. We can list several familiar examples from physics and engineering, biology and economics.

In an internal combustion engine, the higher the temperature differential between the combustion chamber and the environment, the higher the efficiency in transforming heat into work. This is Carnot's Principle, the foundation of thermodynamics. At the same time, it is more expensive to build a combustion chamber that can withstand higher temperature and pressure. Diesel burns at higher temperature than gasoline. This is why the energy efficiency of diesel engine is higher and the cost of building a diesel engine is higher than the cost of building a gasoline engine. In electricity transmission, higher voltage will lower heat loss. At the same time, higher voltage transmission systems are more expensive to build and maintain because the distance from the line to the ground has to be greater to reduce the risk of electric shock. The differential of water levels inside and outside a hydro dam generates electricity. The higher the hydro dam, the more electricity can be generated. At the same time, a higher hydro dam is more costly to build and maintain. Warm blooded animals can run faster than cold blooded animals because their body temperature is maintained at high levels to ensure fast biochemical reactions. But the basic metabolism rates of warm-blooded animals are much higher than those of cold-blooded animals. Shops located near high traffic flows generate high sales volume per unit time. But the rent costs in such locations are also higher. Well trained employees work more efficiently. But employee training is costly. The tradeoff between lower variable cost and higher fixed cost is often not explicitly discussed in the same literature. Our production theory provides a quantitative measure of return and profit at different levels of fixed cost and variable cost under different circumstances.

From our production theory, it can be calculated that when the fixed cost is zero, variable cost is equal to product value. This means that any organisms or organizations need to make fixed investments before earning a positive return. This simple result has broad implications. Thus, any viable organization, whether a company or a country, needs a common fixed investment as a point of departure. This principle has a direct bearing on the question of taxation, among other issues.

In current economic literature, taxes are often described as a type of distortion or imperfection. If this were true, any society that abolishes taxation will remove the distortion and become less imperfect. Over time, such a society would crowd out social systems that collect taxes. However, this has not happened. Montesquieu (1748) observed long ago, "In moderate states, there is a compensation for heavy taxes; it is liberty. In despotic states, there is an equivalent for liberty; it is the modest taxes." From the mainstream economic theory, one might conclude that despotic states are more perfect than moderate states.

In our production theory, taxes are considered as a fixed cost of the whole society. Therefore, lower a tax rate does not automatically mean a better society. The proper level of taxation should be determined by other factors, such as the general living standard of the population, including the physical and social security that make economic endeavors possible.

In neoclassical economics, the market is a very abstract concept.

> The "market" in modern usage is not some physical location. … The market is the nonstate, and thus it can do everything the state can do but with none of the procedures or rules or limitations. … Because the word lacks any observable, regular, consistent meaning, marvelous powers can be assigned. The market establishes Value. It resolves conflict. It ensures Efficiency in the assignment of each factor of production to its most Valued use. … From each according to Supply, to each according to Demand. The market is thus truly a type of God, "wiser and more powerful than the largest computer," … Markets achieve effortlessly exactly what governments fail to achieve by directed effort. (Galbraith, 2008, p. 20)

In our production theory, the market is a concrete concept, with parameters including notably market size. If a market is very small, then its structure will be very simple to reduce fixed cost. If a market is very large, then its structure will be very sophisticated to reduce variable cost. For example, a village market has little formal structure or regulation. But the New York Stock Exchange has very expensive computer systems and highly complex regulatory structures to ensure the smooth flow of a large volume of transactions. In this concrete structure, it is meaningless to discuss whether "the market" is "efficient" or not. Any market that yields a positive return will survive and prosper. Any market that yields negative returns will shrink and disappear.

For an organism or organization, part of fixed cost is used to regulate the internal environment. For example, the human body is regulated at around 37 degrees centigrade. When a person is infected, body temperature is raised, to attack more effectively the infecting bacteria or virus. However, temperature that is too high will damage the brain's information processing capability, which is sensitive to thermal noise (Gisolfi and Mora, 2000). In general, regulation is a compromise between different parts of an organism. When a part of an organism escapes regulation, that part of the organism will grow rapidly. This is called cancer. In the end, the unregulated growth, if it is not stopped, will drain all the resources of the organism and destroy the organism. In human society, if an economic sector gains control of large amount of resources and escapes regulation, becoming a "free market," it will grow rapidly and generate a huge profit for the insiders. But the process will drain resources from the whole society, leading to crisis and collapse. This is what we witnessed in the 2007 - 2009 financial crisis.

Although mainstream economists pay little attention to the biophysical foundation of human society, thinking about biophysical constraints is a very effective way to understand social phenomena (Galbraith, 2008). A biophysical approach puts the physical conditions facing human society at the center of its analysis. The validity of a physical theory of economics is best shown by mathematical expressions derived from the most fundamental properties of life, and consistent with a wide range of patterns observed in both economics and ecology. After all, all physical laws are represented by mathematical formulas. With an analytical theory based on biophysical principles, many philosophical and verbal problems can be turned into scientific and quantitative inquiries. The computability of the mathematical theory will transform biological science, which includes social science as a special case, into an integral part of physics.

This paper is an update from earlier work (Chen, 2005, 2015; Chen and Galbraith, 2011, 2012a, 2012b; Galbraith, 2014). It is structured as follows. Section Two presents the derivation of the production theory. Section Three presents basic results from this theory. Section Four discusses investment decisions in different environments. Section Five explores relations among different parameters that were initially assumed to be independent. Section Six compares our theory to the mainstream neoclassical theory. Section Seven concludes.

2. A mathematical theory of production

The theory described in this section can be applied to both biological and economic systems. For simplicity of exposition, we will use the language of economics. However, the extension to biological systems is straightforward.

The most fundamental property of organisms and organizations is their need to obtain resources from the environment to compensate for the continuous diffusion of resources they engender. This fundamental property can be represented mathematically by lognormal processes, which contain both a growth term and a dissipation term.

Suppose S represents the quantity of resources accumulated by an organism or the unit price of a commodity, r, the rate of resource extraction or the expected rate of change of price and s, the rate of diffusion of resources or the rate of volatility of price change. Then the process of S can be represented by the lognormal process

$$\frac{dS}{S} = r\,dt + \sigma dz\,. \tag{1}$$

where

$dz = \varepsilon\sqrt{dt}, \quad \varepsilon \in N(0,1)$ is a random variable with standard Gaussian distribution

The process (1) is a stochastic process. In this paper, we assume r to be positive and constant. But r can change over time. For example, after a living system dies, r turns negative during its decomposition. Similarly, r turns negative when a firm fails and its assets are dismantled and sold for scrap.

Although a stochastic process will generate many different outcomes over time, we are mostly interested in the average outcomes from such processes. For example, although the movement of individual gas molecules is very volatile, air in a room, which consists of many gas molecules, generates a stable pressure and temperature. We usually study the average outcomes of stochastic processes by looking at the averages of the underlying stochastic variables and their functions. These investigations often transform stochastic processes into their corresponding deterministic equations. For example, heat is a random movement of molecules. Yet the heat process is often studied by using heat equations, a type of deterministic partial differential equation.

Feynman (1948) developed a method of averaging stochastic processes under very general conditions, which is usually called a path integral. Kac (1951) extended Feynman's method into a mapping between stochastic processes and partial differential equations, which was later known as the Feynman-Kac formula. According to the Feynman-Kac formula (Øksendal, 1998, p. 135), if

$$C(t,S) = e^{-qt}E(f(S_t)) \qquad (2)$$

is the expected value of a function of S at time t, discounted at the rate q, then $C(t,S)$ satisfies the following equation

$$\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - qC \qquad (3)$$

with

$$C(0,S) = f(S) \qquad (4)$$

It should be noted that many functions of S satisfy equation (3). The specific property of a particular function is determined by the initial condition (4). This is similar to the Black-Scholes option theory. The Black-Scholes equation is satisfied by any derivative security. It is the end condition at contract maturity that determines the specific property of a particular derivative security.

For an organism or an organization to be viable, the total cost of extracting resources has to be less than the gain from the amount of resources extracted; in economic terms the total cost of operation has to be less than the total revenue. Costs include fixed cost and variable cost. In general, production factors that last for a long time, such as capital equipment, are considered fixed cost while production factors that last for a short time, such as raw materials, are considered variable costs. If employees are on long term contracts, they may be better understood as fixed costs, although in the economic literature, they are usually classified as variable costs. Typically, a lower variable cost system requires a larger investment in fixed costs, though the converse is not necessarily true. Organisms and organizations can adjust their level of fixed and variable costs to achieve a high return on their investment. Intuitively, in a large and stable market, firms will invest heavily in fixed cost to reduce variable cost, thus achieving a higher level of economy of scale. In a small or volatile market, firms will invest less in fixed cost to maintain a high level of flexibility.

Suppose there is a project with a duration that is infinitesimally small. It only has enough time to produce one unit of product. In order to avoid an arbitrage opportunity, if the fixed cost is lower than the value of the product the variable cost should be the difference between the value of the product and the fixed cost. If the fixed cost is higher than the value of this product, there should be no extra variable cost needed for the product. Mathematically, the relation between fixed cost, variable cost and the value of product in this case is the following:

$$C = \max(S - K, 0) \tag{5}$$

Where *S* is the value of the product, *C* is the variable cost and *K* is the fixed cost of the project. When the duration of a project is of finite value *T*, relation (5) can be extended into

$$C(0, S) = \max(S - K, 0) \tag{6}$$

as the initial condition for equation (3). Equation (3) with initial condition (6) can be solved to obtain

$$C = Se^{(r-q)T} N(d_1) - Ke^{-qT} N(d_2) \tag{7}$$

where

$$d_1 = \frac{\ln(S / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

The function *N(x)* is the cumulative probability distribution function for a standardized normal random variable. From (6), the solution of the equation (3) can be interpreted as the variable cost of the project. We will investigate shortly whether the function represented in formula (7) has the common properties of variable cost.

For a given investment problem, different parties may select different discount rates. To simplify our investigation, we will make the discount rate equal to the expected rate of growth. This is to set

$$q = r \tag{8}$$

This choice of discount rate can be understood from two perspectives. From a biological perspective, fast growing organisms also have a high probability of death. In a steady state, the growth rate has to be equal to the death rate. In the biological literature, the discount rate is usually set equal to the growth rate (Stearns, 1992). From the perspective of economics, in option theory, the discount rate is set equal to the risk-free rate. The level of risk of an option contract is represented by implied volatility, which does not necessarily equate with past volatility or future expected volatility. Some people do not agree with the economic logic behind the mathematical derivation of the Black-Scholes equation, which made the risk related discount rate disappear (Treynor, 1996). However, the disappearance of a separate discount rate greatly simplifies our understanding of how option values are related to market variables. From both a biological and economic perspective, this choice of discount rate provides a good starting point.

With q equals r, equation (3) becomes

$$\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC \tag{9}$$

and solution (7) becomes

$$C = SN(d_1) - Ke^{-rT}N(d_2) \tag{10}$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Formula (10) provides an analytical formula for C, variable cost as a function of S, product value, for K, fixed cost, for σ, uncertainty, for T, duration of project, and for r, discount rate of a firm. Similar to physics, the calculated variable cost is the average expected cost of the variable inputs. With this formula, we can calculate how variable cost changes with respect to other major factors in economic and biological activities. Formula (10) takes the same form as the Black-Scholes formula for European call options. But the meanings of parameters in this theory differ from those in option theory.

We now briefly examine the properties of formula (10) as a representation of variable cost. First, when fixed cost is positive, variable cost is always less than the value of the product. No one will invest in a project if the expected variable cost is higher than the product value. Second, when the fixed cost is zero, the expected variable cost is equal to the value of the product. This means that businesses must make a fixed investment before they can expect a profit, just as an organisms must develop a fixed structure before they can extract resources profitably[1]. Third, when fixed costs, *K*, are higher, variable costs, *C*, are lower. Fourth, for the same amount of the fixed cost, when the duration of a project, *T*, is longer, the variable cost is higher. This shows that investment value depreciates with time. Fifth, when risk, s, increases, the variable cost increases. Sixth, when the discount rate falls, variable cost also decreases. This is due to the lower cost of borrowing. All these properties are consistent with our intuitive understanding of, and empirical patterns in, production processes.

After obtaining the formula for the variable cost in production, we can calculate the expected profit and rate of return of an investment. Suppose the volume of output during the project life is *Q*, which is bound by production capacity or market size. During the project's life, we assume the present value of the product to be *S* and the variable cost to be *C*. Then the total present value of the product and the total cost of production are

$$SQ \text{ and } CQ+K \tag{11}$$

respectively. The net present value of the project is

$$QS-(QC+K)=Q(S-C)-K \tag{12}$$

The rate of return of this project can be represented by

$$\frac{QS-(K+QC)}{K+QC} = \frac{QS}{K+QC} -1 \tag{13}$$

It is often convenient to represent *S* as the value of output from a project over one unit of time. If the project lasts for *T* units of time, the net present value of the project is

$$TS-(TC+K)=T(S-C)-K \tag{14}$$

The rate of return of this project can be represented by

$$\frac{TS-(K+TC)}{K+TC} = \frac{TS}{K+TC} -1 \tag{15}$$

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[1] Some do not agree with this statement and provide examples of low fixed cost investment with high profits, such as J. K. Rowling writing Harry Potter books. However, while a small percentage of authors earn high incomes from blockbusters, an average author does not earn a high income, and our results are about the statistical average.

Unlike a mere conceptual framework, this mathematical theory enables us to make quantitative calculations of returns of different projects in differing environments.

Since our theory is very similar to Black-Scholes option theory, we may compare the basic equations of Black-Scholes theory and our theory. The Black-Scholes equation is

$$-\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC \tag{16}$$

The basic equation in our theory is

$$\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC \tag{9}$$

The Black-Scholes equation has a negative sign in front of the time derivative. From a physics perspective, our equation represents a thermodynamic process while the Black-Scholes equation represents a reverse thermodynamic process. From the economic perspective, Black-Scholes equation solves the current price of derivative securities when the future payout is determined; our equation solves the expected variable cost in the future when the current fixed cost is determined. The two theories solve different economic problems.

3. Basic results of the theory

Our theory provides quantitative measurements of how major factors in economic and biological systems affect costs and returns in those systems.

Fixed cost and uncertainty:

By calculating variable costs from (10), we find that, as fixed costs are increased, variable costs decrease rapidly in a low uncertainty environment and change very little in a high uncertainty environment. To put it another way, high fixed cost systems are very sensitive to the change of uncertainty level while low fixed cost systems are not. This is illustrated in Figure 1.

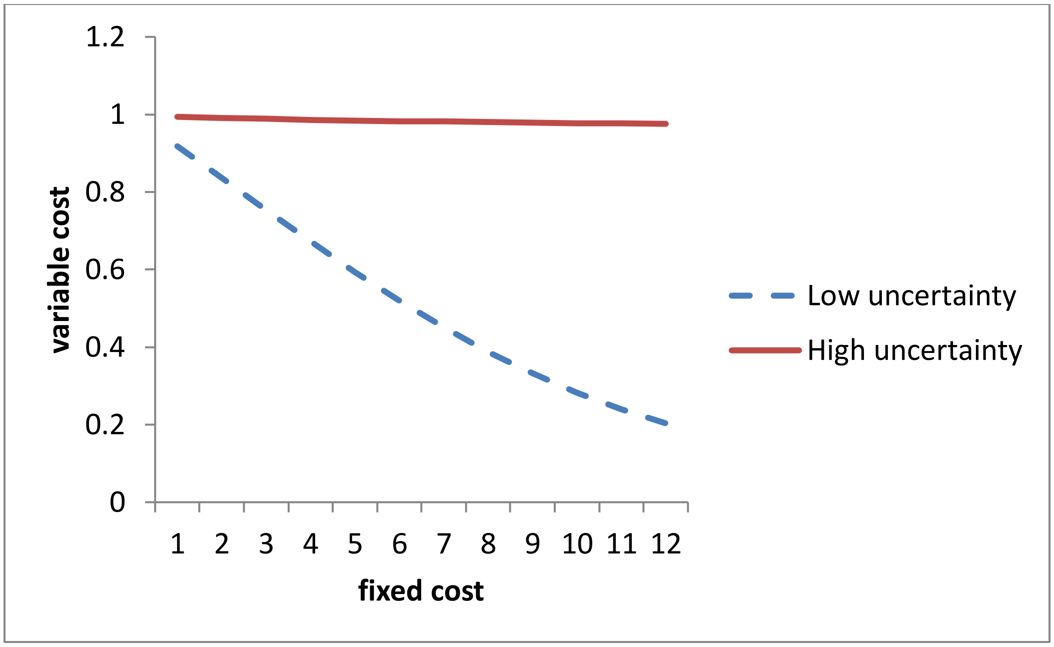

Figure 1. Fixed cost and uncertainty: In a low uncertainty environment, variable cost drops sharply as fixed costs are increased. In a high uncertainty environment, variable costs change little with the level of fixed cost.

The above calculation indicates that systems with higher fixed investment are more effective in a low uncertainty environment and systems with lower fixed investment are more flexible in a high uncertainty environment. Mature industries, such as household supplies, are dominated by established large companies such as Proctor &Gamble while innovative industries, such as information technology, are pioneered by small and new firms. Microsoft, Apple, Netscape, Yahoo, Google, Facebook and countless other innovative businesses are started by one or two individuals. Despite the financial and technical clout of large firms, small firms account for a disproportionately high share of innovative activity (Acs and Audretsch, 1990). Similarly, in scientific research, mature areas are generally dominated by top researchers from elite schools, while scientific revolutions are often initiated by newcomers or outsiders (Kuhn, 1996).

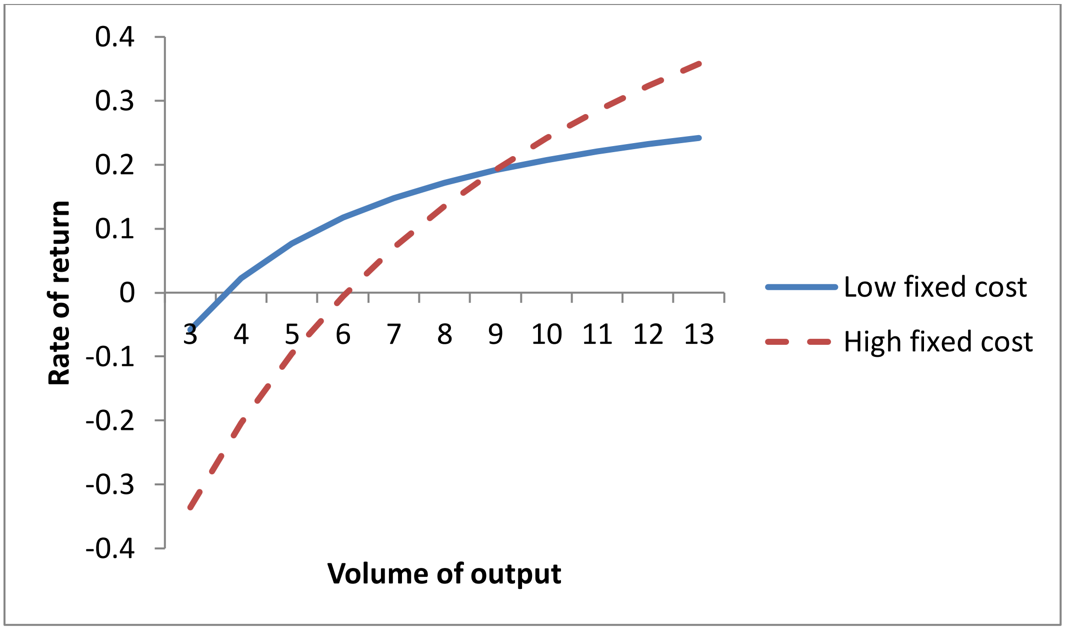

Figure 2. Fixed cost and the volume of output: For a high fixed cost investment, the breakeven market size is higher and the return curve is steeper. The opposite is true for a low fixed cost investment.

Fixed cost and the volume of output or market size:

We now discuss the returns on investment of projects with different fixed costs, with respect to the volume of output or market size. Figure 2 is the graphic representation of (14). In general, higher fixed-cost projects need higher output volumes to break even. At the same time, higher fixed-cost projects, which have lower variable costs in production, earn higher rates of return in large markets.

We can see that the proper level of fixed investment in a project depends on the expectation of uncertainty and the size of the market. When the outlook is stable and the market size is large, projects with high fixed investment earn higher rates of return. When the outlook is uncertain or market size is small, projects with low fixed costs break even more quickly.

In the ecological system, the market size can be understood as the size of the resource base. When resources are abundant, an ecological system can support large, complex organisms (Colinvaux, 1978). Physicists and biologists are often puzzled by the apparent tendency for biological systems to form complex structures, which seems to contradict the second law of thermodynamics (Schneider and Sagan, 2005; Rubí, 2008). However, once we realize that systems with higher fixed cost provide higher returns in resource-rich and stable environments, this evolutionary pattern becomes easy to understand. An example from physiology will highlight this tradeoff:

> An increased oxygen capacity of the blood, caused by the presence of a respiratory pigment, reduces the volume of blood that must be pumped to supply

> oxygen to the tissues. …The higher the oxygen capacity of the blood, the less volume needs to be pumped. There is a trade-off here between the cost of providing the respiratory pigment and the cost of pumping, and the question is, which strategy pays best? It seems that for highly active animals a high oxygen capacity is most important; for slow and sluggish animals it may be more economical to avoid a heavy investment in the synthesis of high concentrations of a respiratory pigment. (Schmidt-Nielson, 1997, p. 120)

For high output systems (highly active animals) investment in fixed cost (respiratory pigment) is favored while for low output systems (slow and sluggish animals) high variable cost (more pumping) is preferred. Pumping is variable cost compared with respiratory pigment because respiratory pigment lasts much longer.

With a volatile commodity market, people become aware of the problem of resource depletion. Many people have advocated the increase of efficiency as a way of reducing energy consumption. Will the increase of efficiency reduce overall resource consumption? Jevons made the following observation more than one hundred years ago.

> It is credibly stated, too, that a manufacturer often spends no more in fuel where it is dear than where it is cheap. But persons will commit a great oversight here if they overlook the cost of improved and complicated engine, is higher than that of a simple one. The question is one of capital against current expenditure. … It is wholly a confusion of ideas to suppose that the economic use of fuel is equivalent to the diminished consumption. The very contrary is the truth. As a rule, new modes of economy will lead to an increase of consumption according to a principle recognized in many parallel instances. (Jevons, 1865 (1965), p. xxxv and p. 140)

Put another way, the improvement of technology is to achieve lower variable cost at the expense of higher fixed cost. Since it takes a larger output for higher fixed cost systems to break even, to earn a positive return the total use of energy has to be higher than before. That is, technology advancement in energy efficiency will increase total energy consumption. Jevons’ statement has stood the test of time. Indeed, the total consumption of energy has kept growing, almost uninterrupted decade after decade for several centuries, along with the continuous efficiency gain of energy conversion (Inhaber, 1997; Smil, 2003; Hall, 2004).

Hybrid cars are an example. Hybrid cars have two engines, one internal combustion engine and one electric engine. This adds to the manufacturing cost (and hence resource consumption) of hybrid cars. If the owner of a hybrid car drives very little, the total resource consumption from a hybrid car is actually higher than a conventional car. Only when a hybrid car is used extensively, burning a lot of fuel, it becomes less wasteful relative to a conventional car. Therefore, the use of a hybrid car, when manufacturing cost is included, guarantees high resource consumption.

Fixed cost and return

We next examine how different levels of fixed investment affect the value of a project. From Figure 3, as the fixed cost of a project is increased, the net present value of the project will increase initially. When the level of fixed cost is at a certain level, its further increase will reduce the value of the project. Eventually the value of the project will become negative. Education is a type of fixed cost in our life. We generally regard education a worthwhile investment. But the majority of people will not pursue Ph.D. degrees.

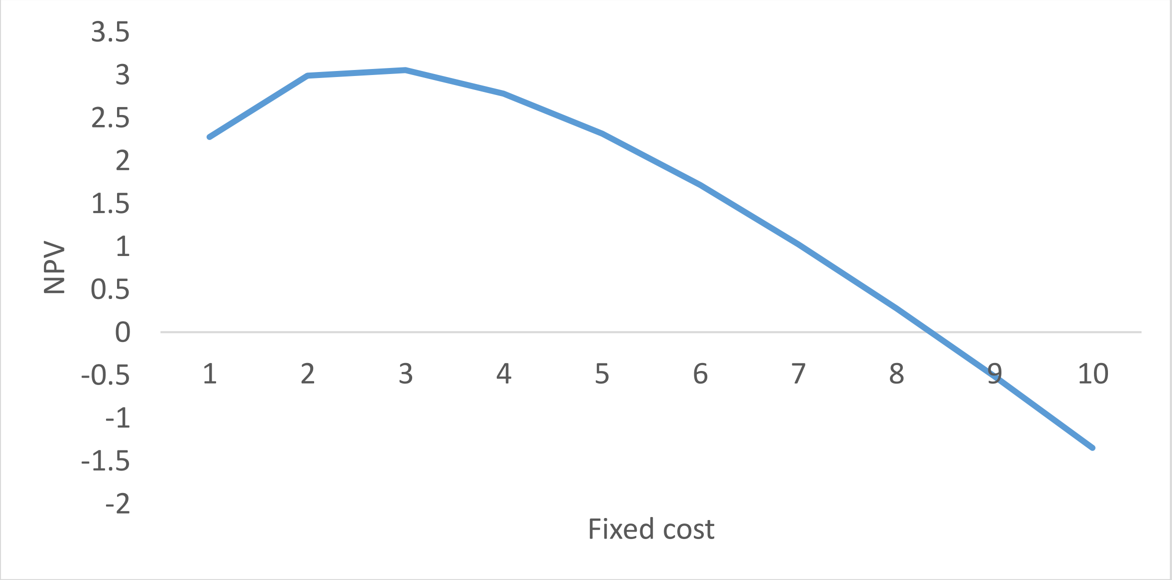

Figure 3. Fixed cost and return:

Duration of the project and return

Next, consider how the duration of projects affects their return. If the duration of a project is too short, we may not be able to recoup the fixed cost invested. If the duration of a project is too long, the variable cost, or the maintenance cost, may become too high. With our mathematical theory, we can make quantitative calculations. The detailed calculation of Formula (14) is illustrated in Figure 4. Similar to Figure 3, as the lifespan of a project is increased, the net present value of the project will increase initially. When the lifespan is at certain level, its further increase will reduce the value of the project. Eventually the value of the project will become negative. This explains why individual life does not go on forever. Instead, it is of higher return for animals to have a finite life span and to produce offspring. The calculation also explains why most businesses fail in the end (Ormerod, 2005).

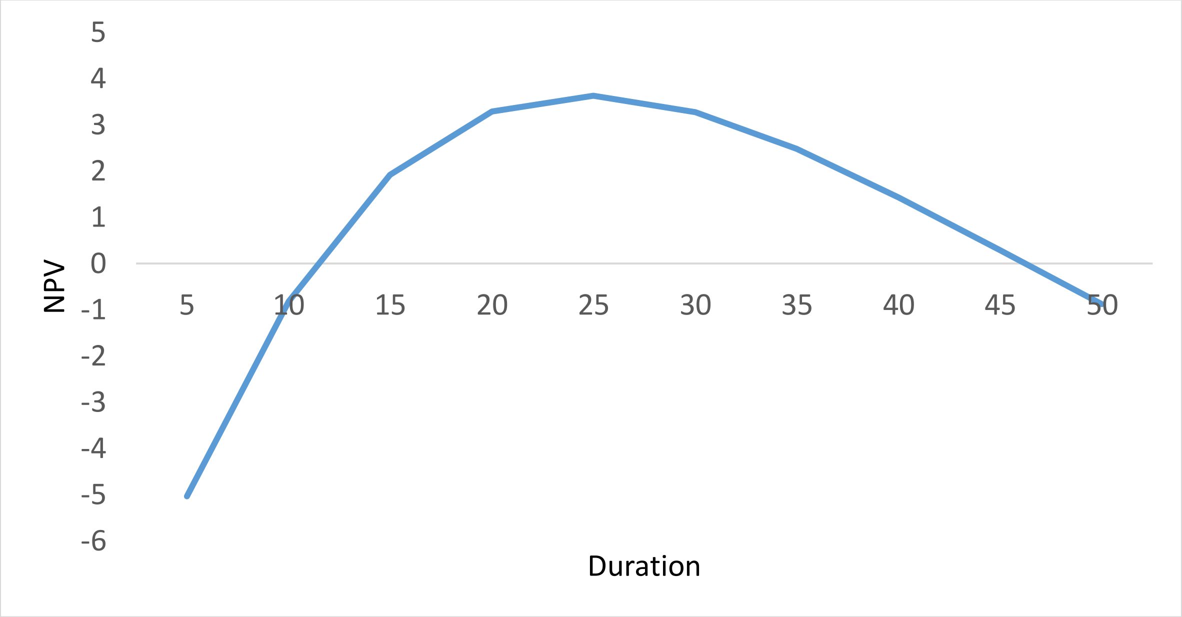

Figure 4. Duration and return

The formula also shows that when fixed costs increase, the duration required for a project to earn a positive return also increases. This suggests that large animals and large projects, which have higher fixed cost, often have longer lives. There is an empirical regularity that animals of larger sizes generally live longer (Whitfield, 2006). The relation between fixed cost and duration can be also applied to human relationships. In child bearing, women spend much more effort than men. Therefore, we would expect women to value long-term relationships while men often seek short-term relations, which is indeed the case most of the time (Pinker, 1997).

Since higher fixed-cost systems have longer life spans than lower fixed-cost systems, the mutation rates of lower fixed-cost systems are faster. This gives lower fixed-cost systems advantages in adapting to change. For example, the coronavirus is much smaller than human beings and can mutate much faster. This makes it difficult for humans to develop natural immune response or to develop drugs to fight effectively against the coronavirus. However, higher animals develop a general strategy in immune systems that has been very effective most of the time. Instead of developing one kind of antibody, our immune systems produce millions of antibodies of different types. It is highly likely that for any kind of bacteria or viruses, there is a suitable antibody to destroy them. This strategy is very effective but very expensive, because our body needs to produce many different antibodies that are useless most of the time. When we are too young, too old or too weak, our bodies don’t have enough energy to produce large amounts of antibodies. That is when we get sick often.

From calculation, when the duration of a project keeps increasing, the return of a project will eventually turn negative. Hence, duration of a project or an organism cannot become infinite. For life to continue, there has to be a systematic way to generate new organisms from old organisms. From earlier calculation, for a system to have a positive return, fixed assets have to be invested first. To achieve this, old organisms have to transfer part of their resources to younger organisms as the seed capital, before the younger organisms can maintain positive return. Therefore, there is a universal necessity of resource transfer from the old generation to the younger generation in the biological world. "Higher" animals, such as mammals, generally provide more investment to each child than "lower" animals, such as fish. In human societies, parents provide their children for some years before they become financially independent. In general, wealthy societies provide more investment to children before they start to compete in the market than poor societies. Similarly in business, new projects are heavily subsidized at their beginning stages by cash flows from profitable mature projects.

Since project life or organism life cannot last forever, resource transfer from organism to organism or from project to project is unavoidable. However, the process of transfer is often the source of many conflicts. Businesses prefer lower tax rates. Educational institutions prefer higher subsidies. Each child wants more attention from its parents. Parents would like to distribute resources more or less evenly among different children. Mature industries, which need little R&D, prefer low tax systems. High tech industries, which rely heavily on universities to provide new technologies, employees and users, strongly advocate government support of new technologies. In good times, financial institutions preach the virtue of free markets. In bad times, the same institutions will remind the public how government support can ensure financial stability of the nation. The amount of resource transfer and the method of resource transfer often define the characteristics of a species or a society.

Fixed cost and discount rate:

The level of fixed cost affects the preference for discount rates. When discount rates are decreased, the variable costs of high fixed cost systems decrease faster than the variable costs of low fixed cost systems (Figure 5). This indicates that high fixed cost systems have more incentive to maintain low discount rates or lending rates. This result helps us understand why prevailing lending rates are different in different places and at different times.

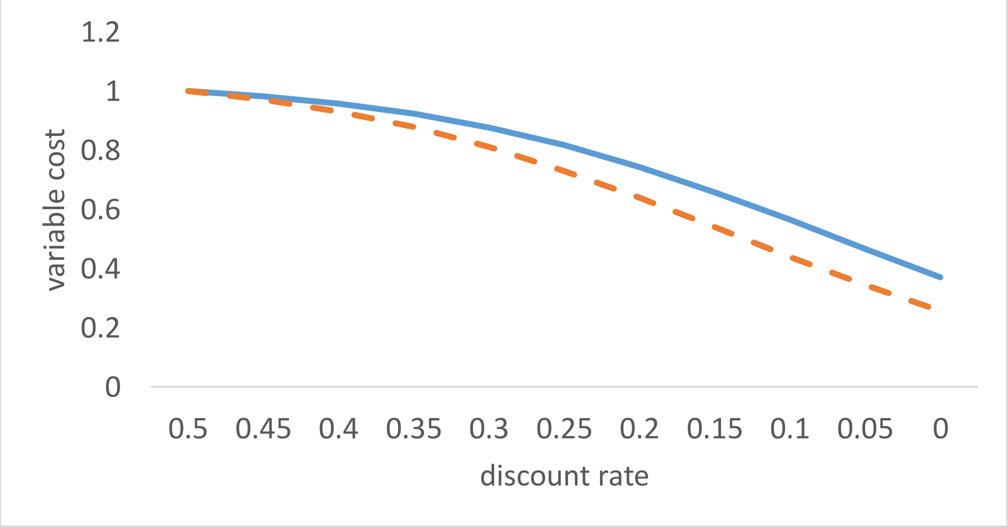

Figure 5. Fixed cost and discount rate: When discount rates are decreased, the variable costs of high fixed-cost systems decrease faster than variable costs of low fixed-cost systems.

In poor countries, lending rates are very high; in wealthy countries, lending rates charged by regular financial institutions, other than unsecured personal loans such as credit card debts, are generally very low. To maintain a low level of lending rates, many credit and legal agencies are needed to inform and enforce standards, which is very costly. As wealthy countries are of high fixed cost, they are willing to put up with the high cost of credit and legal agencies because the efficiency gain from a lower lending rate is higher in high fixed cost systems. In the last several hundred years, there is in general an upward trend in living standards worldwide. There is also a downward trend in interest rates (Newell and Pizer, 2003). This is well-explained by our calculations.

Empirical investigations show that the human mind intuitively understands the relation between the discount rate and different levels of assets. In the field of human psychology, there is an empirical regularity called the "magnitude effect" -- small outcomes are discounted more than large ones. In Thaler's (1981) study, respondents were, on average, indifferent between $15 immediately and $60 in a year, $250 immediately and $350 in a year, and $3000 immediately and $4,000 in a year, implying discount rates of 139%, 34% and 29%, respectively. Since the human mind is an adaptation to the needs of survival and reproduction, evaluating the relation between discount rates and amounts of fixed investment must be a common task in our evolutionary past.

Our understanding about discount rates and fixed costs is similar to an earlier work by Ainslie and Herrnstein (1981):

> The biological value of a low discount rate is limited by its requiring the organism to detect which one of all the events occurring over a preceding period of hours or days led to a particular reinforcer. As the discounting rate falls, the informational load increases. Without substantial discounting, a reinforcer would act with nearly full force not only on the behaviors that immediately preceded it, but also on those that had been emitted in past hours or days. The task of factoring out which behaviors had actually led to reward could exceed the information processing capacity of a species.

Discount rate and project duration:

When the discount rate becomes lower, the variable cost of a project will decrease and profit will increase. Projects with different durations will be affected differently by the reduction of discount rates. Figure 6 presents ratios of profits between projects at low and high discount rates at different levels of project duration. As durations increase, the ratios mostly increase as well. This indicates that projects with longer duration benefit more from the reduction of interest rates. Keynes made a similar argument that as interest increases, the optimal duration of production process is shortened (Keynes, 1936, p. 216).

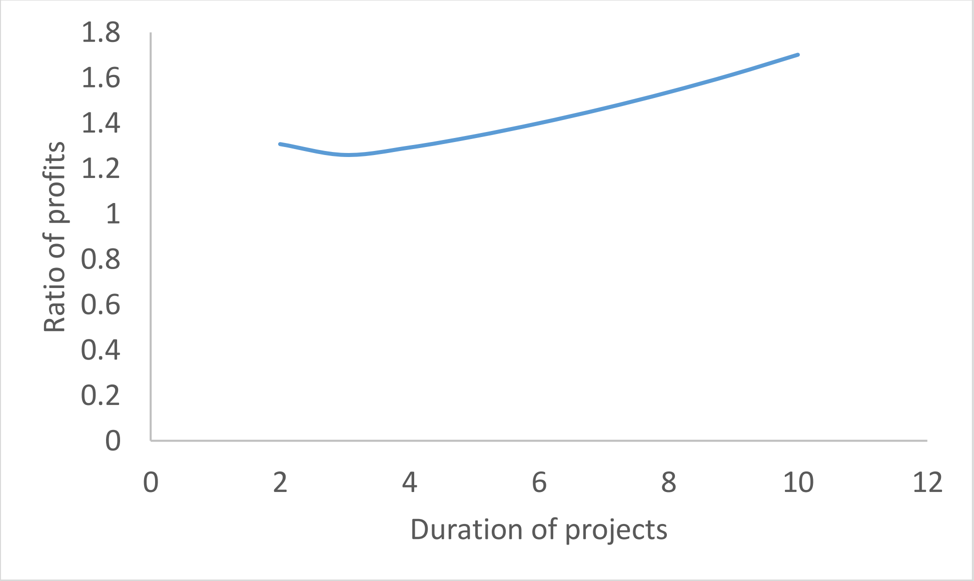

Figure 6. Project duration and discount rate: the ratios of profits between projects at low and high discount rates at different levels of project duration

We can calculate the breakeven point of a project with respect to the project duration and the discount rate. Let us assume that project output per unit of time is one. Formula (14) shows that it requires a lower discount rate to break even when the project duration is lengthened. The calculation is illustrated in Figure 7. Many empirical studies have documented that humans, as well as other animals, often discount long duration events at lower rates than short duration events (Frederick, Loewenstein and O'Donoghue, 2004). This pattern is called hyperbolic discounting. The calculation provides a possible explanation for hyperbolic discounting: since it takes lower discount rates for long duration projects to break even, human mind and also those of other animals will discount long duration projects a lower rates.

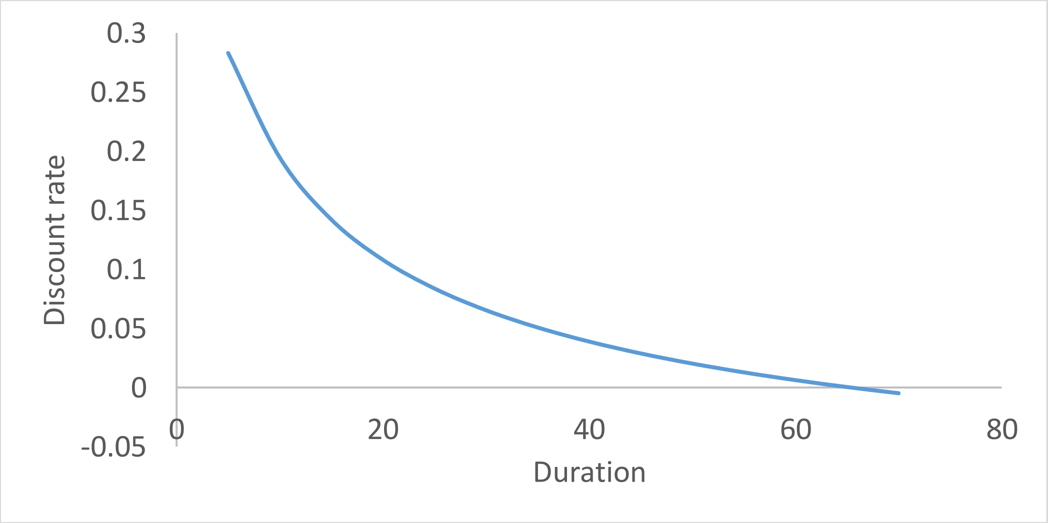

Figure 7. Required discount rate for the project to break even at different project duration: As project duration increases, the required discount rate for a project to break even decreases. This provides a possible explanation for hyperbolic discounting.

In the following, we present more empirical evidence about the inverse relationship between discount rates and durations of a project or span of life. Fecundity, as well as mortality rate, is a proxy for the discount rate (Stearns, 1992). Lane (2002) provides a detailed discussion about the tradeoff between longevity and fecundity in biological systems.

> Notwithstanding difficulties in specifying the maximum lifespan and reproductive potential of animals in the wild, or even in zoos, the answer is an unequivocal yes. With a few exceptions, usually explicable by particular circumstances, there is indeed a strong inverse relationship between fecundity and maximum lifespan. Mice, for example, start breeding at about six weeks old, produce many litters a year, and live for about three years. Domestic cats start breeding at about one year, produce two or three litters annually, and live for about 15 to 20 years. Herbivores usually have one offspring a year and live for 30 to 40 years. The implication is that high fecundity has a cost in terms of survival, and conversely, that investing in long-term survival reduces fecundity.
>
> Do factors that increase lifespan decrease fecundity? There are number of indications that they do. Calorie restriction, for example, in which animals are fed a balanced low-calorie diet, usually increase maximum life span by 30 to 50 per cent, and lower fecundity during the period of dietary restriction. … The rationale in the wild seems clear enough: if food is scarce, unrestrained breeding would threaten the lives of parents as well as offspring. Calorie restriction simulates mild starvation and increase stress-resistance in general. Animals that survive the famine are restored to normal fecundity in times of plenty. But then, if the evolved response to famine is to put life on hold until times of plenty, we would expect to find an inverse relationship between fecundity and survival. (Lane, 2002, p. 229)

Lane went on to provide many more examples on the inverse relation between longevity and fecundity.

In human society, we often use longevity, or duration of human life, as an indicator of the quality of a social environment. At the same time, societies that enjoy a long life span, such as Japan, are often concerned about below replacement fertility. Intuitively, the aging population needs many resources to maintain their health, and this reduces the resources available to support children. Hence, there is a natural tradeoff between longevity and fertility. This result has important policy implications on the balance between resource distribution on longevity and fertility. In a society with below replacement fertility and low immigration, it poses a great challenge to maintain a sustainable society.

Discount rate and uncertainty

Variable cost is an increasing function of the discount rate. When uncertainty is low, variable cost is much lower with a low level of discount rate. When uncertainty is high, variable costs are not very sensitive to the discount rate. Therefore, it is often more effective to reduce discount rate in a stable environment. Figure 8 presents the change of variable costs at different levels of discount rates when levels of uncertainty are low and high. It shows that the reduction of variable cost is more significant at a low uncertainty level. This explains why simpler species, such as algae, or grasses, which have high discount rates, often thrive in highly uncertain environments. It may show why in times of economic crisis low interest rates have little effect on the level of perceived profitability and therefore on activity. This is called "pushing on a string."

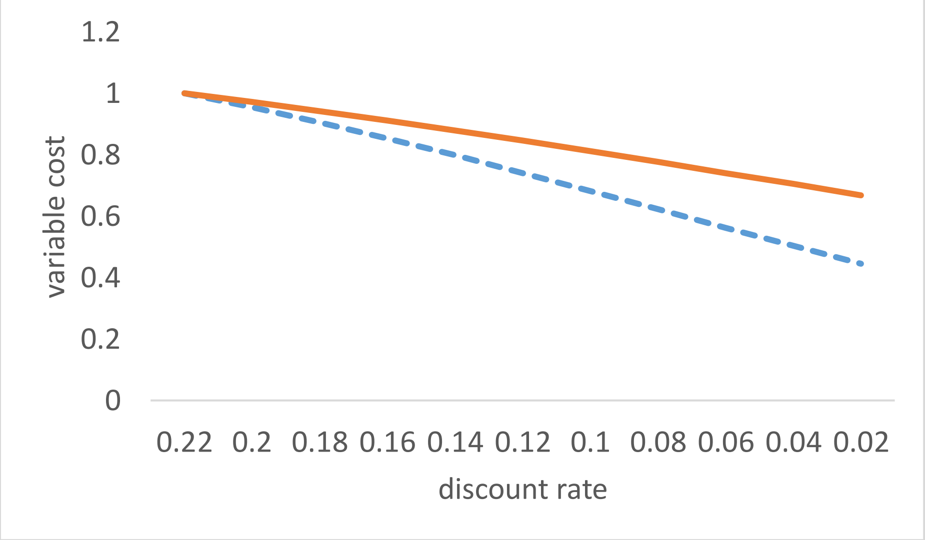

Figure 8. Change of variable costs with discount rate at different levels of uncertainty

This idea about the relationship between discount rate and uncertainty is, by the way, not new. "The same discount curve that is optimally steep for an organism's intelligence in a poorly predictable environment will make him unnecessarily shortsighted in a more predictable environment (Ainslie, 1992, p. 86). Our theory gives this insight a precise quantitative expression.

4. Decision making in different environments

Decision makers will attempt to maximize the value or return of an investment in any given environment. Very often, the discount rate and uncertainty are external environmental factors not controlled by businesses. Businesses therefore choose the level of fixed cost and lifespan of projects to maximize the value or return of a project in that specific environment of uncertainty and discount rates that they confront.

Monetary policies and business cycles

Let the discount rate take two different values at 3% and 10% per annum respectively, while keeping uncertainty unchanged. We will choose the level of fixed cost and lifespan of the project to maximize formula (14), the net present value of the project. The left two columns of Table 1 list the maximization results.

discount rate	0.03	0.1		0.03	0.1
annual output	1	1		1	1
fixed cost	7.1	3.9		7.1	3.9
duration of project	35.7	13.6		35.7	13.6
uncertainty	0.3	0.3		0.8	0.8
variable cost	0.46	0.42		0.97	0.86
NPV	12.3	4.0		-6.2	-2.0

Table 1.

From Table 1, we find that when the discount rate is lower, the amount of fixed investment is larger, investment duration is longer and the net present value is higher. So, investors normally prefer a low interest rate environment. However, the net present values are expected returns calculated at the beginning of a project. The actual returns depend on the future environment. Suppose after the projects are built, the actual level of uncertainty becomes 80% per annum instead of 30% previously expected. We can recalculate the net present values from (14) to find the net present value of the first project, built in the low interest rate environment of 3%, becomes -6.2 billion dollars while the net present value of the second project, built in the high interest rate environment of 10%, becomes -2.0 billion dollars. The right side of the above table lists all the results in the new environment. Both projects suffer losses, but the first project suffers much greater losses. When environmental conditions change, returns on investment in low interest rate environments experience larger fluctuations. In other words, the monetary policy of low interest rates will generate greater business cycles. This theory provides a simple and clear understanding of the relation between the level of interest rate and the magnitude of business cycles.

Stability is destabilizing

Hyman Minsky once said, “Stability is destabilizing”. What does this mean exactly? From our formula, we can obtain a very clear understanding, with an example similar to the one at the beginning of this section. Suppose in two countries, A and B, annual output is 1 billion dollars. Suppose the interest rate is 5% per annum in both countries. Uncertainty rate is 30% per annum in country A and 55% per annum in country B. Decision makers attempt to maximize the net present value of investment project. How much will be the desired fixed costs and how long will the expected project last? What are the net present values of projects in countries A and B?

We attempt to maximize the net present value expression in (14) by changing fixed cost, K and duration, T when uncertainty rates are set at 30% and 55% per annum respectively. The left side of Table 2 lists the calculated results.

uncertainty	0.3	0.55		0.8	0.8
annual output	1	1		1	1
fixed cost	5.8	2.1		5.8	2.1
discount rate	0.05	0.05		0.05	0.05
duration of project	25.3	12.1		25.3	12.1
variable cost	0.44	0.64		0.94	0.82
NPV	8.5	2.3		-4.4	0.0

Table 2.

From Table 2, we find that when uncertainty is lower, the amount of fixed investment is larger, investment duration is longer and the net present value is higher. So investors normally prefer low uncertainty environments. However, the net present values are only the expected returns, calculated at the beginning of a project. The actual returns depend on future conditions. Suppose again that after the projects are built, the actual level of uncertainty becomes 80% per annum in both countries due to circumstances unforeseen by decision makers, such as a global financial crisis. We recalculate (14) to find the new net present values. The net present value of the first project, built in the low uncertainty rate environment, is -4.4 billion dollars while the net present value of the second project, built in the high uncertainty environment, is 0.0 billion dollars. These results are listed on the right side of Table 2. The first project suffers heavy losses, while the second project barely breaks even. When environmental conditions change dramatically, values of investment in formerly stable environment experience large fluctuations. In other words, "stability is destabilizing."

With this analytical theory, simulation is very simple. It enables us to perceive long-term consequences of economic policies and social structures clearly. Detailed discussion of monetary policy and business cycles is presented in Chen (2012, 2015). Many economists and policymakers do sense the long-term implications of their policies. However, without a simple tool to communicate these long-term impacts, most people naturally focus their attention to short term outcomes.

5. Relations among parameters previously assumed independent

So far, we have assumed the parameters in the production processes, fixed cost, duration of the project, discount rate, uncertainty and quantity of output, to be independent variables. But in reality, these parameters often have complex and varying relations. It takes detailed knowledge and deep insight about each system to model these relations well. In the following, we will present some simple attempts to model such relations.

All parameters in our theory, except uncertainty, correspond to directly observable quantities. This is very similar to option pricing theory, in which all parameters, except volatility, correspond to directly observable quantities. In option pricing theory, volatility is often called implied volatility because volatility is implied from the option prices. Similarly in our theory, uncertainty is implied from the expected variable costs. Indeed, the value of uncertainty can involve many factors. The meaning of the "rate of uncertainty" can be very different in different applications.

Economy of scale and the law of diminishing return

All economic systems experience economies of scale and the law of diminishing return at the same time. This can be modelled by setting uncertainty, σ, as an increasing function of the volume of output. Specifically, we can assume

$$\sigma = \sigma_0 + lQ$$

where σ_0 is the base level of uncertainty, Q is the volume of output, and $l > 0$ is a coefficient. Intuitively, when the size of a company increases and the business expands, internal coordination and external marketing become more complex. With this new assumption, we can calculate the rate of return of production from formula (13). The result is presented in Figure 9. The figure shows that the rate of return initially increases with production scale, which is well-known as the economy of scale (Romer, 1986). When the size of the output increases further, the rate of return begins to decline. This is the law of diminishing returns.

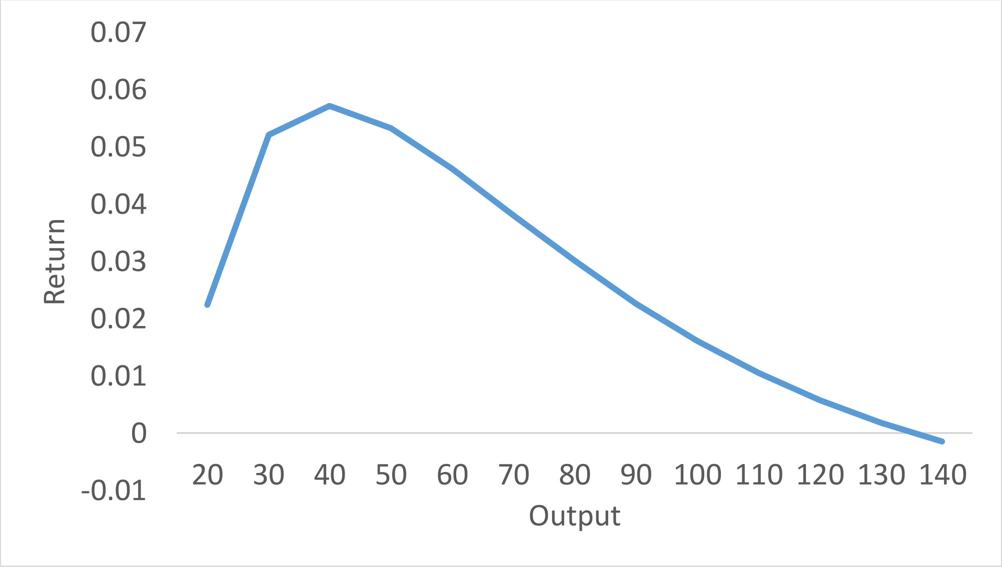

Figure 9. Volume of output and the rate of return: The rate of return of a project with respect to volume of output, when uncertainty is an increasing function of volume of output.

Increasing fixed cost to reduce uncertainty

When the fixed cost of a system increases, the increased fixed cost can often help reduce uncertainty. Many organisms, from stegosaurus to turtles, invest in armor to decrease predation. Air conditioning and heating systems can reduce the uncertainty of temperature in a building. But air conditioning requires an increase in electricity consumption. Insurance can reduce uncertainty of large losses for the policyholders. Yet, insurance premiums must be paid. This pattern can be modeled with uncertainty, σ, as a decreasing function of the fixed cost. Specifically, we can assume

$$\sigma = \sigma_0 + e^{-lK}$$

where σ_0 is the base level of uncertainty, K is the fixed cost and $l > 0$ is a coefficient. Assume the unit product value is 1, discount rate is 5% per annum, and duration of the project is 10 years. Assume σ_0 is 20% per annum and l is 0.2. The calculated rate of return with different levels of fixed cost is shown in Figure 10. When the level of fixed cost is increased, the rate of return increases initially and then declines.

Many decisions involve the spending of fixed cost to reduce uncertainty; this is true of unemployment insurance, old age insurance, medical insurance, and government guarantees to financial institutions. People often take polar positions in debating these issues. A good quantitative model may help us reach compromise among various parties.

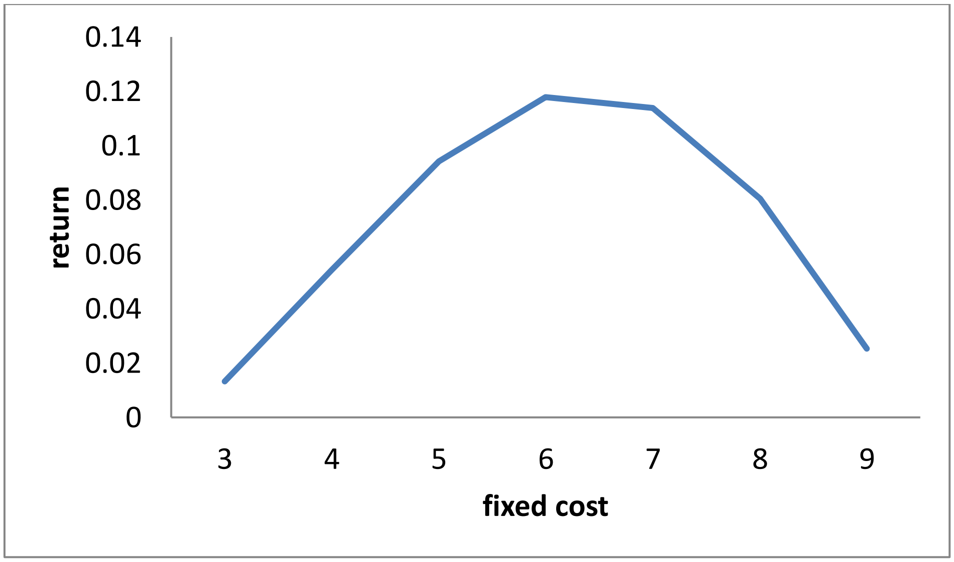

Figure 10. Increasing fixed cost to reduce uncertainty, showing the relation between level of fixed cost investment and rate of return

Resource abundance and investment decisions

One way to represent the relationship between resource abundance and quality is by the level of uncertainty. In physics, the term representing uncertainty in a lognormal process is often called the diffusion rate or the dissipation rate. A higher dissipation rate means that more energy is wasted as heat, and less energy is available to do useful work, indicating a low quality of energy fuels. For example, when a dry cell gets discharged, its internal resistance gradually increases and more energy turns into unusable heat. The quality of the dry cell declines over time. So, the quality of resources can be represented by the (inverse of) uncertainty, or the dissipation rate.

We next model the increase of processing costs for natural resources through the increase of the diffusion rate, to understand how these costs affect the structure and size of economic systems. From the calculation of (10), when the diffusion rate is higher, the variable cost becomes higher. Intuitively, higher diffusion rates mean more effort is needed to process a given amount of resources. Specifically, the level of diffusion will be modelled as:

$$\sigma = \sigma_0 + lQ$$

σ_0 is the base level of diffusion, which corresponds to the lowest cost in production when resource of highest quality is used. Q is the total output of economy and $l > 0$ is a coefficient. The value of l represents the abundance of low-cost resources; when a low-cost resource is abundant, l is small. An increase of output will not increase processing costs substantially. When the low-cost resource is scarce, l is large. An increase of output will require high-cost resources, which increases the processing cost substantially.

For simplicity, we set $S = 1$, $r = 0.1$, $T = 15$ and $\sigma_0 = 0.4$. We let l take the values of 0.0025, 0.005 and 0.01 to represent different levels of resource abundance. By maximizing formula (13) with respect to the fixed cost and volume of output at different values of l, we obtain the highest possible rate of return from investment projects in different environments. When $l = 0.0025$, projects obtain the highest possible rate of return of 28% when the fixed cost is 9.5 and market size is 56. When $l = 0.005$, projects obtain the highest possible rate of return of 15% when fixed cost is 4.5 and market size is 33. When $l = 0.01$, projects obtain highest possible rate of return of 6% when the fixed cost is 1.7 and market size is 20. Figure 11 displays the rates of return with respect to different sizes of output by three different projects with different fixed costs, corresponding to different levels of resource abundance.

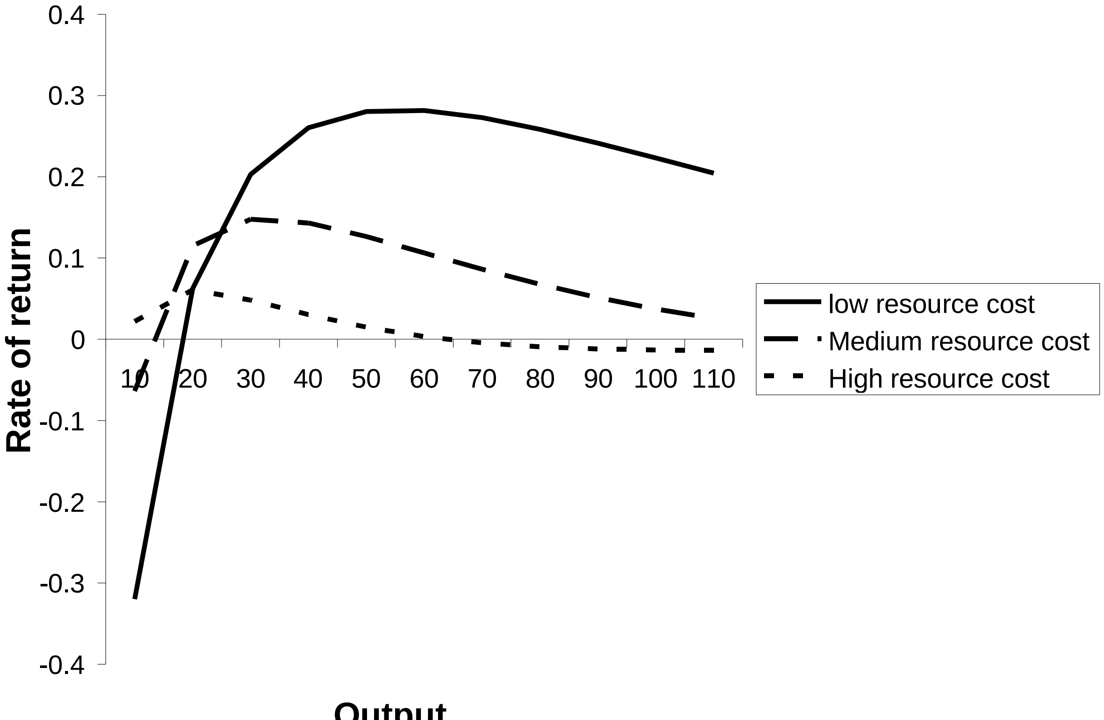

Figure 11. Resource abundance, fixed cost, market size and rates of returns: The rates of return with respect to different output scales for three different projects with different fixed costs corresponding to different levels of resource abundance.

As the figure shows, when resources are abundant and cheap there is an economic incentive to increase fixed cost and market size. As a result, the rate of resource consumption is high. When the low-cost resources are gradually depleted, the return from the same high fixed cost system will decline gradually. To understand the precise relation between resource abundance, economic structure and rate of return, we calculate the rates of return for the same high fixed cost system, which provides the highest possible rate of return, at $l = 0.0025$, when $l = 0.005$ and 0.01. Figure 12 displays the rates of return with respect to different output scales by the same high fixed-cost production system, at different levels of resource abundance. Compared with Figure 11, when the cost of resource production is moderately higher, represented by $l = 0.005$, the difference of return between the existing high–fixed-cost production system and the potential optimal production system is not large. Furthermore, developing new projects with different levels of fixed cost may require new skills and equipment. Therefore, the incentive to change is small. But when low-cost resources are further depleted, represented by $l = 0.01$, rates of returns of the high fixed cost production system turn negative at all levels of output, as shown in Figure 12. Reduction of fixed cost and output size are then required to restore economic activities to positive returns.

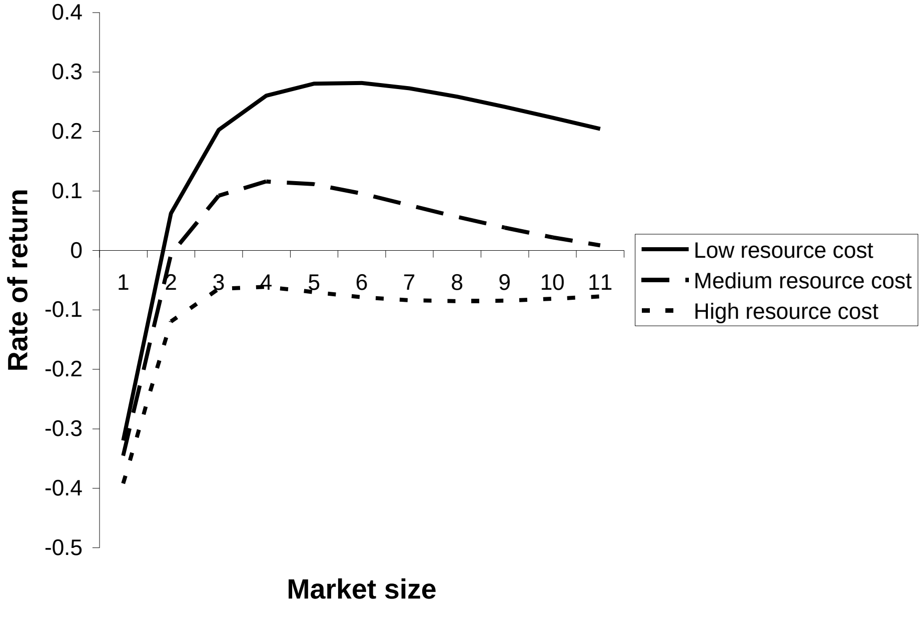

Figure 12. Resource abundance and rates of returns: The rates of return with respect to different output scales for the same high-fixed-cost production system corresponding to different levels of resource abundance.

Since it takes a long time and great effort to adjust institutional structures, it will be very helpful to estimate the state of resource abundance today and in the near future. In a now classic paper titled The End of Cheap Oil, Campbell and Laherrere (1998), after carefully examining the data on oil exploration and production, concluded “What our society does face, and soon, is the end of the abundant and cheap oil on which all industrial nations depend.” This does not mean that oil will disappear and become unavailable. It means that the cost of obtaining fresh supplies will rise, and the high-fixed-cost systems built on the premise of cheap oil will become unprofitable.

6. A Comparison with Neoclassical Economic Theory

Since its birth, the foundation or “assumptions” of neoclassical economic theory has been criticized for its lack of relevance to reality. To this, Friedman replied:

> In so far as a theory can be said to have “assumptions” at all, and in so far as their “realism” can be judged independently of the validity of predictions, the relation between the significance of a theory and the “realism” of its “assumptions” is almost the opposite of that suggested by the view under criticism. Truly important and significant hypotheses will be found to have “assumptions” that are wildly inaccurate descriptive representations of reality, and in general, the more significant the theory, the more unrealistic the assumptions (in this sense). The reason is simple. A hypothesis is important if it “explains” much by little, that is, if it abstracts the common and crucial elements from the mass of complex and detailed circumstances surrounding the phenomena to be explained and permits valid predictions on the basis of them alone. To be important, therefore, a hypothesis must be descriptively false in its assumptions; it takes account of, and accounts for, none of the many other attendant circumstances, since its very success shows them to be irrelevant for the phenomena to be explained. (Friedman, 1953, p. 16)

He further challenged:

> As we have seen, criticism of this type is largely beside the point unless supplemented by evidence that a hypothesis in one or another of these respects from the theory being criticized yield better predictions for as wide a range of phenomena. (Friedman, 1953, p. 31)

The prediction power of neoclassical theory is very poor. Many phenomena that are not consistent with theories are labelled as “imperfect”. For example, from Modigliani and Miller (1958) theory, in a perfect market, capital structure is irrelevant. Since capital structure is relevant in reality, the real market is imperfect. There are many similar terms, such as “imperfect information”, “imperfect contract”, “imperfect competition”, “inefficient property right”, “market failure”, “government failure”, “externality”. Before discussing these imperfections, we briefly review the idea of imperfection in old astronomy.

Ancient people had long observed that stars moved in perfect harmony in the sky. Several planets, however, moved in irregular trajectories. It was thought that this was caused by the imperfectness of the planets. There were many elaborate theories why the planets were imperfect. Kepler, however, derived that all planets moved in perfect elliptic orbits. This story tells us that “imperfection of the world” often reflects imperfection of the theory that is used to understand the world.

Our analytical production theory offers a unified understanding of various “imperfections” or “externalities”. In the following, we will briefly compare our production theory and neoclassical economic theory.

Consistency with physical and biological theories

Neoclassical economics was founded around 1870 by Jevons, Walras and others, who believed that economics should be built on a sound physical foundation. Since the dominant platform of physics in Jevons and Walras’ time was Newtonian mechanics, it was natural for them to adopt this platform. However, theories derived from rational mechanics often do not offer good explanation to economic behaviors. Gradually, explicit identification with physics disappears while analogies between physics and economics are frequently mentioned. The following quote from Samuelson’s Nobel lecture is quite representative:

> There is really nothing more pathetic than to have an economist or a retired engineer try to force analogies between the concepts of physics and the concepts of economics. How many dreary papers have I had to referee in which the author is looking for something that corresponds to entropy or to one or another form of energy.

In the very next paragraph, however, Samuelson found an analogy himself.

> However, if you look upon the monopolistic firm hiring ninety-nine inputs as an example of a maximum system, you can connect up its structural relations with those that prevail for an entropy-maximizing thermodynamic system. Pressure and volume, and for that matter absolute temperature and entropy, have to each other the same conjugate or dualistic relation that the wage rate has to labor or the land rent has to acres of land.

Mirowski observed, “The key to the comprehension of Samuelson’s meteoric rise in the economics profession was his knack for evoking all the outward trapping and ornament of science without ever coming to grips with the actual content or implications of physical theory for his neoclassical economics” (Mirowski, 1989, p. 383).

Life systems are non-equilibrium thermodynamic systems. The current dominant economic theory is general equilibrium theorem. Social systems are a special case of living systems. When a theory about a special case is inconsistent with general foundation, either the general foundation or the special theory is wrong. So far, economists have not challenged the validity of the non-equilibrium thermodynamic theory of life systems. This theory shows that an analytical theory of economics can be directly derived from basic physical and biological principles. By this, it establishes social sciences as an integral part of physical and biological sciences.

A comparison with the Cobb-Douglas production functions

Production functions, such as Cobb-Douglas production function, form the fundamental blocks in neoclassical economic theory. Cobb-Douglas function takes the form

$$Y = AK^{\alpha}L^{1-\alpha}$$

where Y, L and K denote output, labor (variable cost) and capital (fixed cost) respectively. Solow had made following comment about the production function:

> I have never thought of the macroeconomic production function as a rigorously justifiable concept. In my mind it is either an illuminating parable, or else a mere device for handling data, to be used so long as it gives good empirical results, and to be discarded as soon as it doesn’t, or as something better comes along. (Solow, 1966, p. 1259)

The form and parameters of Cobb-Douglas function are given without rigorous justification. A, the coefficient in Cobb-Douglas function, “has been called, among other things, ‘technical change’, ‘total factor productivity’, ‘the residual’ and ‘the measure of our ignorance’” (Blaug, 1980, p. 465).

To gain further understanding about the Cobb Douglas production function, we will perform some calculations of this production function. Let A, the coefficient, take the value of 2. Let capital and labor inputs sum to unity. Let alpha take the value from 0.1 to 0.9. For each case of alpha, let capital take the value from 0.1 to 0.9, while labor takes the value from 0.9 to 0.1, correspondingly. The output values of different scenarios are listed in the following table.

Capital Alpha	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	1.44	1.39	1.29	1.15	1.00	0.83	0.65	0.46	0.25

0.2	1.16	1.21	1.18	1.11	1.00	0.87	0.71	0.53	0.31
0.3	0.93	1.06	1.09	1.06	1.00	0.90	0.77	0.61	0.39
0.4	0.75	0.92	1.00	1.02	1.00	0.94	0.84	0.70	0.48
0.5	0.60	0.80	0.92	0.98	1.00	0.98	0.92	0.80	0.60
0.6	0.48	0.70	0.84	0.94	1.00	1.02	1.00	0.92	0.75
0.7	0.39	0.61	0.77	0.90	1.00	1.06	1.09	1.06	0.93
0.8	0.31	0.53	0.71	0.87	1.00	1.11	1.18	1.21	1.16
0.9	0.25	0.46	0.65	0.83	1.00	1.15	1.29	1.39	1.44

From this calculation, when alpha approaches zero or one, the output reaches a maximum, with A, the coefficient, held constant. This means that pure labor or pure capital are the optimal production mode. Apparently, this is not consistent with empirical patterns. One might argue that A, the coefficient, is not constant with different production modes. But then, we would need to designate A as a function of the production mode, not as a constant.

From the calculation, when the ratio of capital to labor is equal to the ratio of alpha to (1 – alpha), output reaches a maximum. In other words, there is really no flexibility in the proportion of capital and labor for any production mode. To survive in a competitive market, one has to follow the predetermined proportions of capital and labor, given a certain ratio of the production function coefficients.

In an input-output model, one would assume the input is consumed in the process of producing output. However, in standard literature, capital K is retained from one period ot the next, albeit with a reduction of depreciation. (Nobel Foundation, 2018, p. 9) If that is the case, it would be advantageous to use as much capital as possible in each period. One might argue capital is long term. However, if capital is long term, its benefits should be released over the long term, not discharged in every unit of time.

In this theory, labor seems to be independent. Its supply is given exogenously. But the labor supply is not independent. When the demand for education level is high, fertility rate is low. This means that an increase in the share of capital, including “human capital,” will depress labor input over the long term.

The Cobb-Douglas production function has little relevance to real economic activities. A recent meta-analysis of research on Cobb Douglas function concludes that "the weight of evidence accumulated in the empirical literature emphatically rejects the Cobb-Douglas specification." (Gechert, et al, 2021)

One might argue that there is no alternative. But there is one. The analytical production theory presented here is derived rigorously from the fundamental property of life systems. It gives simple and clear results of returns to investment under different market conditions.

Optimality vs. tradeoff

Optimization theory holds the central position in neoclassical economics. Paul Samuelson's Nobel Lecture is titled *Maximum Principles in Analytical Economics*. Friedman (1953) tried to reconcile the maximization principle with evolutionary theory. He stated:

> Confidence in the maximization-of-return hypothesis is justified by evidence of a very different character. … unless the behavior of businessmen in some way or other approximated behavior consistent with the maximization of returns, it seems unlikely that they would remain in business for long. Let the apparent immediate determinant of business behavior be anything at all --- habitual reaction, random chance, or whatnot. Whenever this determinant happens to lead to behavior consistent with rational and maximization of returns, the business will prosper and acquire resources with which to expand; whenever it does not, the business tend to lose resources and can be kept in existence only with addition of resources from outside. The process of "natural selection" thus helps to validate the hypothesis --- or rather, given natural selection, acceptance of the hypothesis can be based largely on the judgment that it summarized appropriately the conditions for survival. (Friedman, 1953, p. 22)

We offer an example of project investment to illustrate the problem with Friedman's argument. Assume the relevant parameters are unit value of the product to be one million, discount rate to be 4%, diffusion to be 40%, duration of the project to be 30 years and market size to be 150 over the project's life. It can be calculated, from Formula (12), that a project with a fixed cost of 25 million dollars will be optimal. However, if any parameter changes, the optimal value of fixed cost investment will change as well. For example, if diffusion increases to 60%, the optimal value of fixed investment will fall to 11 million. Since fixed cost is spent or committed at the beginning of the project while other parameters may change over the course of project life, it is impossible to determine optimality in advance. Furthermore, higher fixed cost systems, which are often the winners of earlier market competition, suffer more from the increase of uncertainty. This means that long term survival is not necessarily consistent with short term optimization.

Earlier, we have shown that systems with higher fixed costs earn higher rates of return in large markets and stable environments than those with lower fixed costs. These systems may appear superior. However, the performance of high fixed cost systems deteriorates in high volatile environments. The main theme of economic and biological evolution is the tradeoff between competitiveness of high fixed cost systems in a stable environment and flexibility of low fixed cost systems in a volatile environment. Biologists haven't found a universally applicable measure of fitness (Stearns, 1992, p. 33). Our theory shows that there does not exist such a measure. For the same reason, there will not exist a universally applicable measure of optimality.

Is marginal cost equal to marginal revenue?

Traditional economic theory suggests that companies will keep increasing output until the marginal cost of the product is equal to its marginal revenue (Friedman, 1953, p. 16). Empirical evidence shows that companies generally charge a substantial price mark up on their products. This analytical theory offers a simple and clear understanding about price markup. For example, if a software is targeted to sophisticated users, its interface can be simple, which reduces development cost, and the sales effort can be small, which reduces variable cost. If the software developer considers increasing its market by targeting general users, the interface of the software needs to be very intuitive with many help facilities, which increases development cost and sales effort, and after-sales service can be substantial for less sophisticated users, which increases variable cost. Since the increase of market size often involves both the increase of variable cost and fixed cost, most projects are designed for marginal cost to be much lower than the product value, so as to maximize potential profit.

To keep increasing output until marginal cost equals marginal revenue often means that the company may have to enter difficult areas, which will have repercussions on its earlier units. For example, when employees in a WalMart store in Quebec decided to unionize, WalMart closed down that store even though that store would remain profitable. To keep a unionized store open would have affected the margin of other stores, whose staff would then attempt to unionize as well. As discussed earlier, the rate of return depends not only on the market size, but also on other factors. If the increase of market size requires an increase of fixed costs and a reduction of flexibility, companies have to consider the total effect on long term profitability.

7. Concluding Remarks

Many pioneering works apply physical and evolutionary ideas to economic theory. These works generally use ordinary differential equations to describe economic activities (Chen, 1987). As biophysical and economic activities are thermodynamic processes, we expect thermodynamic equations, which are partial differential equations, to describe economic activities more accurately than conventional expressions commonly used in economics.

In this paper, we have presented an analytical theory of production by solving a thermodynamic equation. The generality of this theory is a consequence of the generality of physical laws, which apply equally to physical systems, biological systems and economic systems. This allows us to develop a unified production theory that can be applied to many different fields. Historically, some economic principles in physics, such as principle of least action and maximum entropy principle (Jaynes, 1957), have been very fruitful in providing unified foundations to very diverse areas of investigation. The production theory presented in this work provides a unified understanding for a wide range of problems in economics and biology.

This theory has been applied to project investment, corporate finance, trade and migration, resource and social structures, language and cultures, evolutionary and institutional economics, fiscal and monetary policies, business cycles, firm size and competitions, software development economics and other problems (Chen, 2012, 2015;

Chen and Galbraith, 2011, 2012a, 2012b; Liu, Kong, Chen, 2015). However, much more work needs to be done to provide a more accurate and detailed understanding of the implications of our theory for economic and social behavior.

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